

Wavelet and Fourier Transforms in Health Monitoring of Embedded Structures

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ABSTRACT: Embedded structures are widely encountered in construction, such as anti-slide piles in a slope and reinforced concrete frames in a building. The embedded structures are vulnerable subjected to seismic effects. After the seismic load, the integrity of the embedded structures is weakened but the damage is unlikely to be directly observed by engineers; as a result, damage detection of an embedded structure is critical. In this study, the identification of the structural damage is investigated with Wavelet and Fourier transforms. With these two approaches, the structural signal in different frequency domain can be analyzed. The advantages and disadvantages of these two approaches in identifying the damage locations are compared. An outstanding advantage of the Wavelet transform is that it is able to identify the damage location, which makes this approach attractive for engineering practice. This advantage is exemplified by a cantilever beam finite element model.

Structure health monitoring (SHM) is an important way to evaluate the structure condition in aerospace, civil and hydropower engineering. The main task in SHM field is damage detection, especially in damage location. This is even important for an embedded structure in soil subjected to various uncertain loads resulting in unclear location of damages (e.g., an anti-sliding piles for slope stabilization that subjected to seismic loads). To date, the vibration-based SHM methods have received increasing attention for its convenience and economy (Yan et al., 2007). According to whether the model parameters are required, vibration-based methods could be roughly divided into two categories: model-based method and model-free method. For the former, it detects the damage by comparing

the model parameters between intact and cracked structure. The most common model parameters include natural frequency (Sampaio et al., 1999), mode shape (Pandey et al., 1991) and structural damping (Seyedpoor, 2012). Model-free methods could be deemed as a kind of pattern recognition, because the background knowledge, such as mechanical vibration and structural dynamics, is not required. The typical model-free methods include signal processing methods (e.g. wavelet transform (Su et al., 2018) and empirical mode decomposition (Yu et al., 2005)) and intelligent algorithms (e.g. artificial neural network and genetic algorithm (Anijdan et al., 2006)).

In this study, two typical signal processing methods, namely, Fourier transform and wavelet transform, are applied. Fourier transform is

widely applied in analyzing signal frequency components, such as natural frequency. Generally, the structure damage could be detected from the shift of natural frequency. Hence, natural frequency based on Fourier transform is often applied in model-based SHM methods (see Salawu, 1997). Wavelet transform, as a model-free method, attracts lots of attention as well. Many researches in SHM field have been conducted, which are briefly summarized in Table 1.

In order to compare their performance in detecting damage locations, Fourier transform

and wavelet transform method are first applied in two simple self-generated signals, and their advantages and disadvantages in damage location are compared. Then, the finite element models of an intact cantilever beam and a cracked cantilever beam are built, respectively. The finite element analysis (FEA) displacement data with pressure load are obtained. Finally, the time-frequency characteristics of the FEA displacement curve are analyzed with wavelet transform and Fourier transform. Wavelet transform performs superior over Fourier transform in detecting damage location.

Table 1: Summary of wavelet transform in SHM.

References	Structure type	Damage sources
Mardasi et al. (2018)	Beams and plates	Crack
Su et al. (2018)	3-storey & 8-storey steel frame	Earthquake load
Ashory et al. (2018)	Composite laminates plates	Delamination
Yang and Oyadiji (2017)	Multi-layer structure	Internal defects
Su and Huang (2017)	8-storey steel frame	Earthquake load
Wu and Wang (2011)	Aluminum beam	Crack
Taha et al. (2006)	Bridge	Traffic dynamic load
Melhem and Kim (2003)	Concrete road	Traffic dynamic load
Okafor and Dutta (2000)	Beam	Crack
Hou et al. (2000)	Simple structure model	Breaking spring

1. METHODOLOGY

1.1. Fourier transform

As a most common tool in signal processing, Fourier transform converts time-domain to frequency-domain effectively. For signal $S_n = \{S_0, S_1, S_2, \dots, S_{N-1}\}$, its Discrete Fourier Transform F_m (DFT) is

$$F_m = \sum_{n=0}^{N-1} S_n e^{-i \frac{2\pi}{N} nm}, m = 0, 1, 2 \dots N-1 \quad (1)$$

where n and m represents time and Fourier frequency, respectively.

1.2. Wavelet transform

Wavelet transform is applied widely in signal processing for its advantages that it has

great resolution in both time-domain and frequency-domain. When $\psi(t)$ satisfies Eq. (2),

$$\int_R \frac{|\overline{\psi(\omega)}|^2}{|\omega|} d\omega < \infty \quad (2)$$

it calls wavelet basis, or mother wavelet. $\overline{\psi(\omega)}$ is the Fourier transform of $\psi(t)$.

For signal $S(t)$, its Continuous Wavelet transform (CWT) is

$$W_S(a, b) = \int_R S(t) \psi_{a,b}(t) dt \quad (3)$$

where $\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$ ($a > 0$) is the wavelet basis. a is the scale parameter; and b is the localization parameter.

2. SOME BASIC SIGNALS

2.1. Rectangular wave

The rectangular wave is generated by Eq. (4). And, its independent variable t ranges from $[0, 2]$. Setting the sample frequency as 1000 Hz, the rectangular wave is shown as Figure 1. Its DFT and CWT are shown as Figures 2(a) and (b), respectively. For CWT, the wavelet basis is Haar (Taha Et al., 2006), and the decomposition level is 100.

$$y = \begin{cases} 1 & (0 \leq t \leq 1) \\ -1 & (1 < t \leq 2) \end{cases} \quad (4)$$

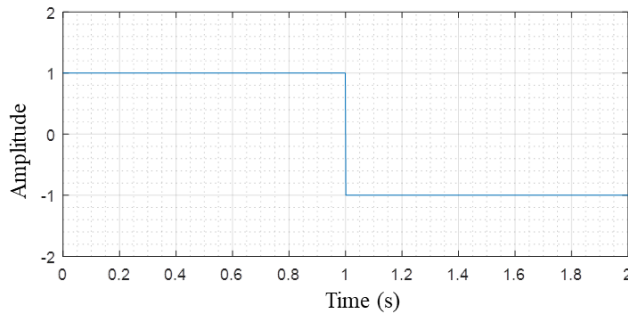


Figure 1: Rectangular wave.

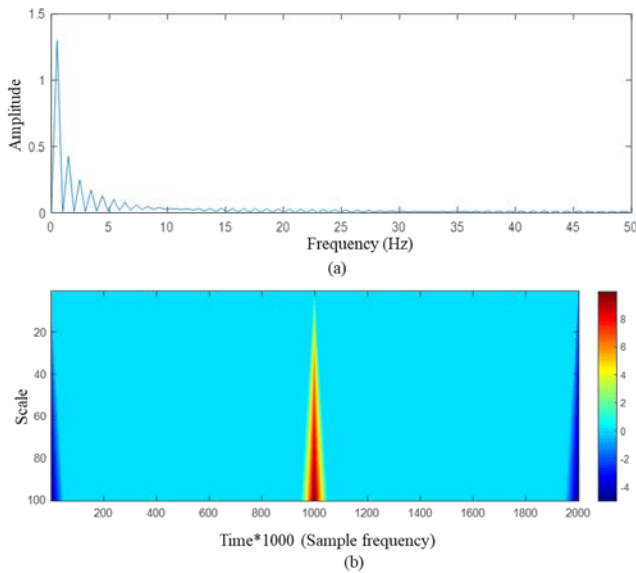


Figure 2: (a) DFT curve of rectangular wave, (b) CWT image of rectangular wave.

Figure 2 shows that the signal variation in time=1 could be detected by CWT effectively, but not by DFT curve. This is determined by

DFT characteristic. Compared with CWT image that obtains both time-domain and frequency-domain information, DFT curve only keeps the frequency-domain information. Hence, without control group, DFT is unable to detect the signal variation in time-domain. This is a significant advantage for CWT in detect time-domain variation over DFT.

2.2. White noise with pulse

To analyze the sensitivity of the methods in variation detection, this study analyzes different amplitude white noise with a certain pulse signal. The pulse amplitude is 10 in time=0.5 s. The white noise amplitude ranges are set as $[-1, 1]$, $[-2, 2]$, $[-3, 3]$ and $[-4, 4]$, respectively. Also, to analyze DFT in variation detection, the DFT results between white noise with pulse and without pulse are compared. The signals are shown as Figure 3 (sample frequency = 1000 Hz). Its DFTs and CWTs are shown as Figures 4 and 5, respectively. For CWT, the mother wavelet is Haar; the decomposition level is 100.

Figure 4 shows that: (1) the DFT frequency overall distribution could be affected as a local pulse added in white noise signal. Especially, the pulse amplitude is much higher than noise as shown in Figures 4 (a) and (b). The environment effects are unavoidable in engineering monitoring. Hence, when DFT is applied to detect natural frequency shifts in SHM, the noise contamination should be considered. (2) Even setting control group, DFT curve is too mess to detect the natural frequency shifts in Figure 4. Because DFT applies infinite harmonic waves to fit a signal essentially, this results in its inherent drawback in analysis non-periodic and instability jump signal, such as blasting vibration signal and seismic wave.

From Figure 5, CWT detects the signal variation at time=0.5 s effectively, when the white noise amplitude ranges are equal or less to $[-3, 3]$ (as Figures 5(a), (b) and (c)). Although some inferences exist in CWT images, such as arrow A in Figure 5(b) and arrow B, C and D in Figure 5(c), the CWT images in pulse location contains a wedge shape obviously compared with

other positions. But when the amplitude range is $[-4, 4]$ (as Figure 5(d)), CWT could not detect the variation location directly. CWT variation detection sensitivity obviously affects by white noise in this case. To analyze the effects of noises, variation features of signals, such as variation size, variation width and the smooth characteristic of the curve, should be considered. This study only analyzes the effects of variation size with white noise. To quantify the variation

size, this study defines it as:

$$\text{variation size} = \frac{\text{pulse amplitude}}{\text{white noise amplitude}}$$
 as such, when the white noise amplitude range is $[-2, 2]$,

$$\text{variation size} = \frac{\text{pulse amplitude}}{\text{white noise amplitude}} = \frac{10}{2} = 5$$
 . Based on this definition, when the variation size is greater than 3.3, CWT could detect the variation effectively in this case.

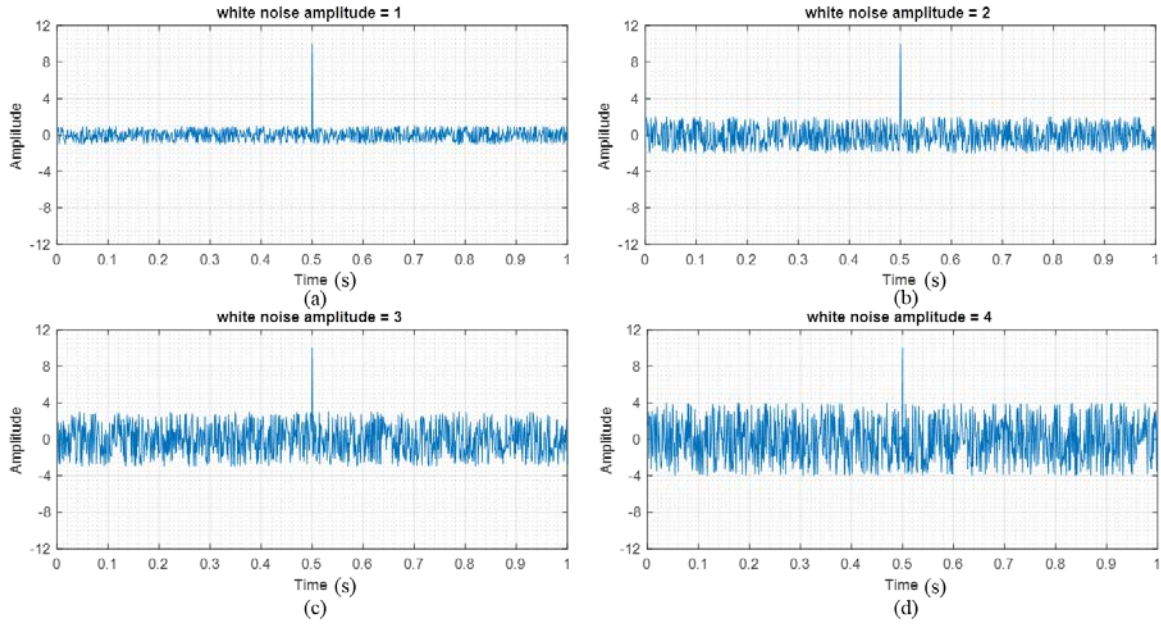


Figure 3: Different white noise amplitude with different amplitudes of pulse.

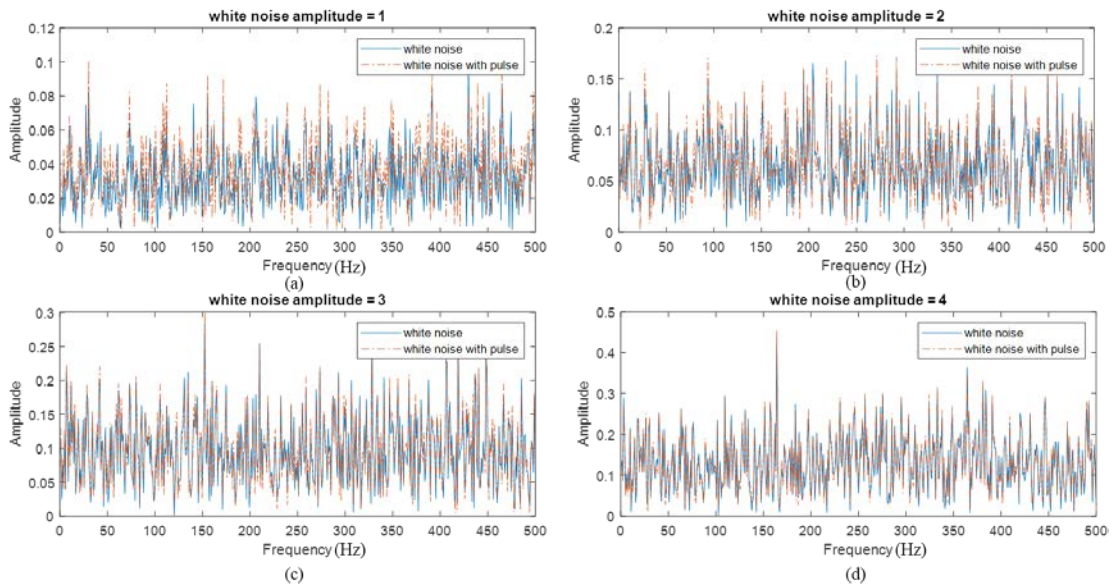


Figure 4: DFT curves of noise amplitude with with different amplitudes of pulse.

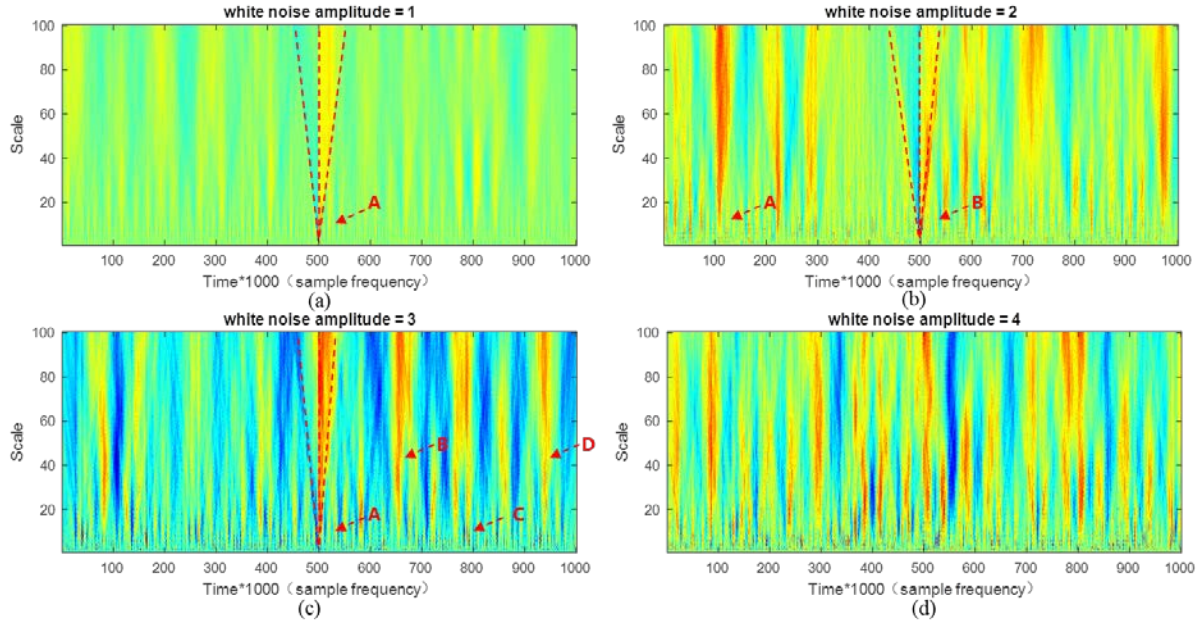


Figure 5: CWT images of noise amplitude with different amplitudes of pulse.

3. NUMERICAL SIMULATION

The finite element method is employed to simulate the deflection of an intact beam and a cracked cantilever beam. These two beams are the same except the crack. The left surface of beam is fixed at the Y-Z plane of coordinates. The beam material is elastic with Young's Modulus = 2.4 GPa and passion ratio = 0.2. The size of beam is set as 2000 mm × 200 mm × 200 mm. For the cracked beam, along the length of beam, a crack with the depth of 10 mm is located from 800 mm to 810 mm on the lower side. To accurately reveal the stress/strain singularity at crack position, a high density mesh is applied. The total element numbers of the model are 79800. The meshed cracked beam is shown as Figure 6. A uniform distributed static downwards load of 5 MPa is applied at the upper of the cantilever beam along the Y-axis. The deflection of cracked beams along the X direction is shown in Figure 7.

Displacement curve is one of the most common signals in engineering monitoring. Hence, displacement data of the Y direction in the middle line of upper surface are recorded. To amplify the variation size of curve, displacement relative to the previous point is applied, which

are shown in Figure 8. The DFT and CWT of the displacement signals are shown as Figures 9 and 10, respectively. For CWT, mother wavelet chooses Sym1 for its good peculiarity in orthogonality and bicomact set; the decomposition level is 100.

From Figure 9, the main frequency of cracked beam drops dramatically compared with the intact beam. Generally, the Fourier main frequency is deemed as the natural frequency of the structure. Hence, in this FEA, the natural frequency of cracked structure decreases than the intact. Also, the frequency distribution is fluctuated in cracked beam, i.e. cracked beam contains multi-order natural frequency. Based on these phenomena, DFT could indicate early damage warning. However, DFT is unable to identify the crack location of structures.

From Figure 10, CWT could detect the damage location at around 800 mm easily. Moreover, the control group is not required. Hence, wavelet transform is more effective and efficient in detecting damage location, especially for those difficult in monitoring structure parameters, such as embedded structures (e.g. an anti-sliding piles for slope stabilization).

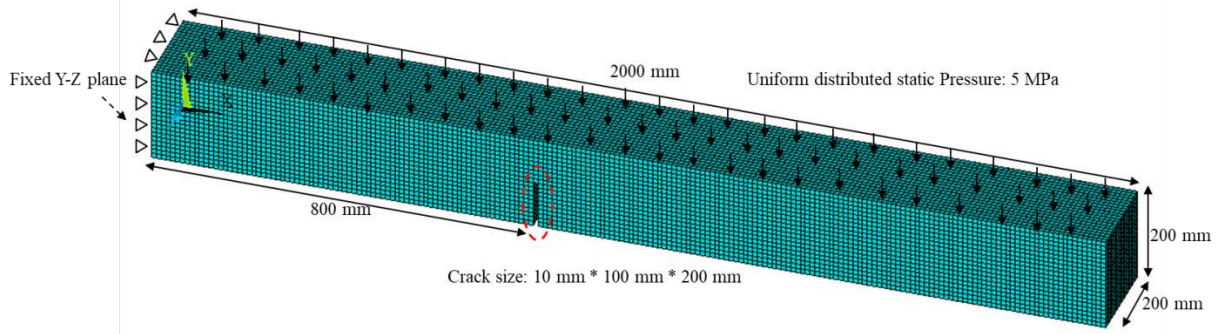


Figure 6: Geometric size, mesh size and boundary conditions of the cracked cantilever beam model.

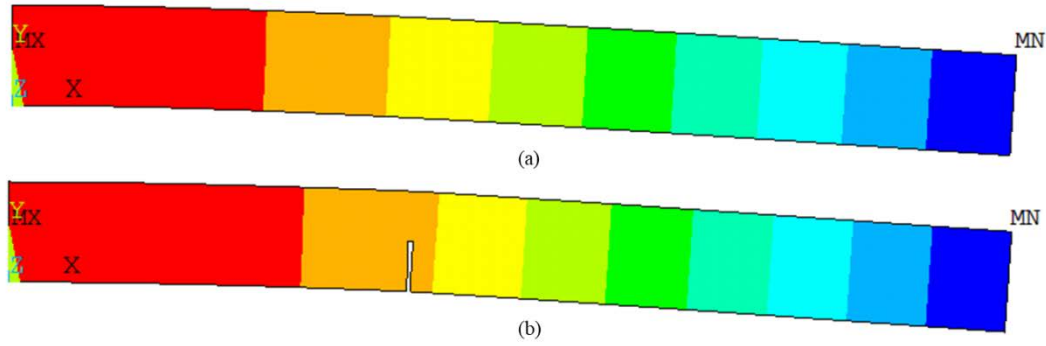


Figure 7: Cracked cantilever beam deflection in X direction of (a) intact beam and (b) cracked beam.

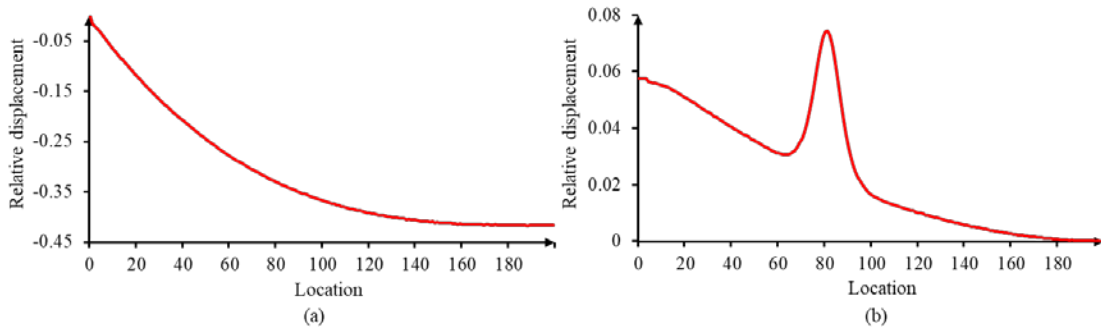


Figure 8: Location-relative displacement curve of (a) Intact beam and (b) cracked beam.

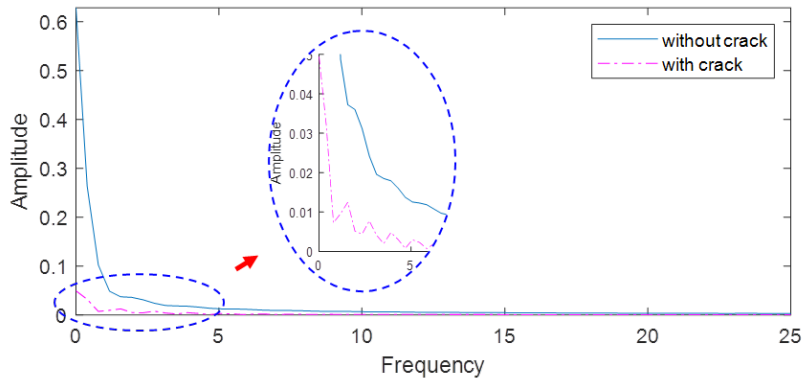


Figure 9: Location-displacement DFT curve.

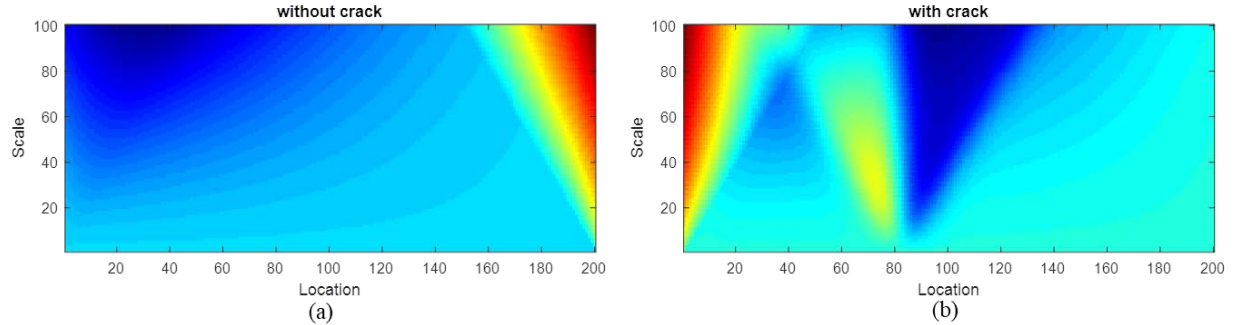


Figure 10: Location-displacement CWT images.

4. CONCLUSION

This study compared the performance of Fourier transform and wavelet transform method in structure health monitoring. Fourier transform is widely applied in engineering practices for its simple theory and clear physical sense, such as Fourier main frequency as natural frequency; however, it also contains some drawbacks in structure health monitoring: (1) it requires control group in damage detection; (2) it could not detect the damage location, because it only contains the frequency-domain information and cannot show the time-domain information of signals; (3) its frequency overall distribution is easily affected by local noise; (4) it is not appropriate for non-periodic and instability engineering signal, because Fourier transform uses infinite harmonic waves to fit a signal essentially. The short-time Fourier transform solves parts of the question about time-domain information, but it is still not precise in many cases. Wavelet transform has better resolutions in both time-domain and frequency-domain. It could detect the damage location merely based the monitoring data. The control group is not required. As a result, wavelet transform is more effective and efficient in detecting damage location, especially for those difficult in monitoring structure parameters, such as an embedded structure (e.g. an anti-sliding piles for slope stabilization).

In using Wavelet transform for structure health monitoring, the signal feature (e.g. variation size, variation width and the smooth

characteristic of the curve) would affect the analysis accuracy. This study only analyzes the effects of variation size in wavelet transform with a simple case. More research in these fields is required for its more wide application in engineering practices. Also, structural health monitoring using wavelet transform is driven by monitoring data, which effectively avoids the complicate theoretical calculation. This feature provides a new idea for the real-time monitoring in structural health monitoring.

5. Acknowledgement

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